MATHEMATICAL PHYSICS

One textbook plus lecture notes permitted.
NO integral tables except that attached.

Do All Problems

1. Evaluate the integrals

(a) $$\int_0^\infty \frac{x^2 dx}{x^4 + 1}$$ (10 points)

(b) $$\int_0^\infty \frac{\ell nx \; dx}{x^2 + 4}$$ (10 points)

2. Find an exact traveling wave solution to the partial differential equation

$$\frac{\partial^2 y}{\partial x^2} - y \frac{\partial y}{\partial t} = 0 \quad .$$

The wave travels in the positive x-direction with speed v and satisfies the conditions

$$y = 0 \qquad \text{for } x = t = 0$$

$$y \to 1 \quad \text{as } x \to \infty \text{ for all finite } t.$$ (20 points)

3. (a) Solve the integral equation

$$f(x) = e^{-x} + \lambda \int_0^\infty dy \sin xy \, f(y) \quad .$$ (16 points)

(b) Under what circumstances does your solution break down? (4 points)

4. (a) Find the function $y(x)$ that gives an extremum value to the integral

$$I = \int_0^1 (y'^2 + y)dx$$

subject to the boundary conditions

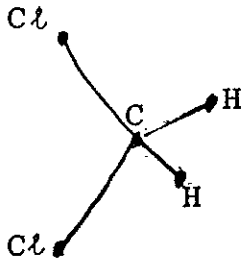
$$y(0) = 0 \quad , \quad y(1) = \frac{1}{4} \quad . \qquad \text{(15 points)}$$

4(b)

(b) What is the nature of the extremum (maximum or minimum) and what is the extremum value of I?

(5 points)

5. The molecule $CH_2C\ell_2$ has the geometrical arrangement shown in the diagram. The atomic groups CH_2 and $C\,C\ell_2$ form isoceles triangles perpendicular to one another. Consider the group of the symmetry operations of this molecule.



(a) What are the symmetry operations? (4 points)

(b) What is the order of the group? (4 points)

(c) How many classes are there? (4 points)

(d) How many irreducible representations are there and what are their dimensionalities? (4 points)

(e) Construct the character table (4 points)

Some Elementary Integrals

$$\int \sin x\,dx = -\cos x \qquad \int \cos x\,dx = \sin x$$

$$\int \tan x\,dx = -\log \cos x \qquad \int \cot x\,dx = \log \sin x$$

$$\int e^x dx = e^x \qquad \int \log x\,dx = x\log x - x$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) \text{ or } -\frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{a^2-x^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) \text{ or } \frac{1}{2a}\log\frac{a+x}{a-x}$$

$$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) \text{ or } -\cos^{-1}\left(\frac{x}{a}\right)$$

$$\int \frac{dx}{\sqrt{x^2\pm a^2}} = \log\left(x + \sqrt{x^2 \pm a^2}\right)$$